University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 212C QM Spring 2023
Assignment 5

Due 11:00am Wednesday, May 10, 2023

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1. **Heat capacity of empty space.** What is the heat capacity of empty space?

   For the purposes of this problem you may ignore all degrees of freedom besides the electromagnetic field. We are particularly interested in the dependence on temperature.

   Compare your calculation to HW02 problem 3d.

2. **Further evidence for the clumping tendencies of bosons.**

   Consider again the model of a 1d crystalline solid that we discussed in class: It consists of N point masses, coupled to their neighbors:

$$\mathbf{H}_0 = \sum_{n=1}^{N} \left(\frac{\mathbf{p}^2}{2m} + \frac{1}{2}\kappa \left(\mathbf{q}_n - \mathbf{q}_{n-1}\right)^2 \right) = \sum_{\{k\}} \hbar\omega_k \left(\mathbf{a}_k^\dagger \mathbf{a}_k + \frac{1}{2} \right) \quad . \tag{1}$$

   Assume periodic boundary conditions $\mathbf{q}_n = \mathbf{q}_{n+N}$, so that the allowed wavenumbers are

$$\{k\} \equiv \{k_j = \frac{2\pi}{Na} j, \quad j = 1, 2 \ldots N\} \ .$$

   Consider a state with two phonons defined by

$$|k_1, k_2\rangle \equiv \mathbf{a}_{k_1}^\dagger \mathbf{a}_{k_2}^\dagger |0\rangle \ .$$

   (a) In the state $|k_1, k_2\rangle$, what is the probability of finding two phonons at the location x_1?

   Do this problem both using a first-quantized point of view and using the algebra of creation and annihilation operators. Make sure your answers agree!

   [Warning: the statement of this problem is deceptively simple.]

   (b) Make sure your probabilities add up to one.

   (c) Compare your result to the answer that would obtain if the particles were distinguishable (and occupied the same two single-particle states). Do bosons clump?

   (d) Does the story change if $k_1 = k_2$?

(e) Bonus problem: for two fermions in the state $|k_1, k_2\rangle$, what is the probability of finding one at x_1 and one at x_2? Check that your probabilities add to one. (It is possible to do this part in parallel with the others.)

3. [Bonus problem] Describe the outcome of the intensity interferometry (Hanbury-Brown and Twiss) experiment for beams of fermions.

4. [Bonus problem] Consider free fermions with single-particle Hamiltonian

$$h = t\sum_n |n\rangle\langle n+1| + h.c. + \sum_n V_n |n\rangle\langle n|.$$

(a) For the case without an external potential, $V_n = 0$, numerically evaluate the single-particle Green's function

$$G(n,m) \equiv \langle\Phi_0|\, \psi_n^\dagger \psi_m \,|\Phi_0\rangle$$

in the groundstate. Plot it as a function of the separation between the two points.

(b) Now add a *random* potential V_n. Choose each V_n independently from a Gaussian distribution with width v. How does this affect the Green's function? What happens if you average $G(n,m)$ over v for fixed $n-m$.