

University of California at San Diego – Department of Physics – Prof. John McGreevy  
**Physics 215C QFT Spring 2026**  
**Assignment 6**

Due 11:59pm Monday, May 18, 2026

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1. **Gross-Neveu model.** This problem uses *very* similar steps to the analysis of the  $O(n)$  model, but leads to very different physical conclusions. I include it here to emphasize the many applications of this method. It also illustrates the manipulations we did in describing the BCS phenomenon. And it's an application of fermionic path integrals,

Consider the partition function for an  $N$ -vector of fermionic spinor fields in  $D$  dimensions:

$$Z = \int [d\psi d\bar{\psi}] e^{iS[\psi]}, \quad S[\vec{\psi}] = \int d^D x \left( \bar{\psi}^a \mathbf{i} \not{\partial} \psi^a - \frac{g}{N} (\bar{\psi}^a \psi^a)^2 \right).$$

- (a) At the free fixed point, what is the dimension of the coupling  $g$  as a function of the number of spacetime dimensions  $D$ ? Show that it is classically marginal in  $D = 2$ , so that this action is (classically) scale invariant.
  - (b) We will show that this model in  $D = 2$  exhibits dimensional transmutation in the form of a dynamically generated mass gap. Here are the steps: first use the Hubbard-Stratonovich trick to replace  $\psi^4$  by  $\sigma\psi^2 + \sigma^2$  in the action, where  $\sigma$  is a scalar field. Then integrate out the  $\psi$  fields. Find the saddle point equation for  $\sigma$ ; argue that the saddle point dominates the integral for large  $N$ . Find a translation invariant saddle point. Plug the saddle point configuration of  $\sigma$  back into the action for  $\psi$  and describe the resulting dynamics.
2. **Right-handed neutrinos.**

Consider adding a right-handed singlet (under all gauge groups) neutrino  $N_R$  to the Standard Model. It may have a majorana mass  $M$ ; and it may have a coupling  $g_\nu$  to leptons, so that all the dimension  $\leq 4$  operators are

$$\mathcal{L}_N = \bar{N}_R \mathbf{i} \not{\partial} N_R - \frac{M}{2} \bar{N}_R^c N_R - \frac{M}{2} \bar{N}_R N_R^c + (g_\nu \bar{N}_R H_i^T L_j \epsilon^{ij} + h.c.)$$

where  $N_R^c = C (\bar{N}_R)^T$  is the the charge conjugate field,  $C = \mathbf{i} \gamma_2 \gamma_0$  (in the Dirac representation),  $H$  is the Higgs doublet,  $L$  is the left-handed lepton doublet, containing  $\nu_L$  and  $e_L$ . Take the mass  $M$  to be large compared to the electroweak

scale. Integrate out the right-handed neutrinos at tree level. [Hint: you may find it useful to work in terms of the Majorana field

$$N \equiv N_R + N_R^c$$

which satisfies  $N = N^c$ .]

Show that the leading term in the expansion in  $1/M$  is a dimension-5 operator made of Standard Model fields. Explain the consequences of this operator for neutrino physics, assuming a vacuum expectation value for the Higgs field.

Place a bound on  $M$  assuming that the observed neutrinos have masses  $m_\nu < 0.5$  eV.