University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2020
Assignment 8

Due 12:30pm Wednesday, March 11, 2020

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1. **Brain-warmer.** Why is $\int d^dx \vec{\nabla}\phi_<(x) \cdot \vec{\nabla}\phi_>(x) = 0$?

2. **Rotation invariance as an emergent symmetry.**

   Give an RG analysis which explains why the critical behavior of lattice magnets (which do not have continuous spatial rotation symmetry) can be described by rotation-invariant field theories.

   To be more precise about what I am asking: consider a hypercubic lattice, and a magnet with an $\mathsf{O}(n)$ symmetry, so that there is an n-component order parameter. As in the problem on HW 8, analyze what perturbations of the rotation-symmetric action preserve lattice rotations but not continuous rotations, and decide what are their scaling dimensions at the Wilson-Fisher fixed point.

   Note that I am asking about *spatial* rotation symmetry, as opposed to rotations in spin space. Please note that a priori the spin rotation symmetry

   $$S(r)^a \to R^{ab} S^b(r), \ R \in \mathsf{O}(n)$$

   is completely independent of the spatial rotation symmetry

   $$S^a\left(r^i\right) \to S^a\left(\Lambda^{ij} r^j\right), \ \Lambda \in \mathsf{O}(d).$$

   Spin-orbit couplings break the product of these two groups down to a diagonal subgroup; such couplings are present in Lorentz-invariant field theories, and in lattice models involving large-Z atoms, but are often negligible.

3. **Order parameter exponent at the Wilson-Fisher fixed point.** In lecture we outlined the computation of η using position space diagrams. Find the coefficient c in $\eta = c\epsilon^2 + \mathcal{O}(\epsilon^3)$ as a function of n at the $\mathsf{O}(n)$ Wilson-Fisher fixed point. Check that the factors of r_2 drop out.

   [Hint: The answer for the Ising model ($n = 1$) is $\eta = \frac{\epsilon^2}{54}$.]

4. **OPE.** Consider the Gaussian fixed point with $\mathsf{O}(n)$ symmetry. Compute the OPE coefficients for the operators $\mathcal{O}_2 \equiv :\phi_a\phi_a:$, $\mathcal{O}_4 \equiv :(\phi_a\phi_a)^2:$, and the identity operator (here $a = 1..n$ and the repeated index is summed). Use this information to compute the beta function, find the Wilson-Fisher fixed point and the correlation length critical exponent ν there.