University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239 Topology from Physics Winter 2021 Assignment 10

Due 5pm Friday March 19, 2021 (if you want to hand it in)

Thanks in advance for following the guidelines on hw01. Please ask me by email if you have any trouble.

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1. **Quantum double algebra exercises.** [Bonus problem]

   (a) Check that the operators A_v and B_p in the quantum double Hamiltonian are projectors.

   (b) Check that

   $$L^h_+ T^g_+ = T^{hg}_+ L^h_+, \qquad L^h_- T^g_+ = T^{ghg^{-1}}_+ L^h_- \tag{1}$$

   and find the analogous relations for L_+T_- and L_-T_-.

   (c) Following Kitaev, define quantum double operators

   $$A^h_s, B^h_s$$

   associated to a *site*, a vertex and an attached plaquette. Show that they satisfy the *quantum double* (or Drinfeld double) algebra:

   $$A^h A^g = A^{hg}, \quad B^h B^g = \delta_{h,g} B^h, \quad A^g B^h = B^{ghg^{-1}} A^g.$$

2. **Ribbon operators.** [Bonus problem]

   (a) Check that the ribbon operators I wrote down in lecture commute with the quantum double hamiltonian except at the endpoints. Therefore they create and transport the excitations.

   (b) Check that the ribbon operators written down by Kitaev commute with the quantum double hamiltonian except at the endpoints. Therefore they create and transport the excitations.

   (c) [Super-bonus problem] For the cubic lattice quantum double model, write down a membrane operator that creates a pure flux (closed) string excitation at its boundary. This is the 3d-generalization of the string operator for the flux excitation in the 2d toric code. The operator should commute with the Hamiltonian everywhere but the boundary of the membrane.