Attempts to realize toric code system from Rydberg Blockade atoms.

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This paper shows a possible way to experimentally realize the topologically ordered system in cold atom system named Rydberg Blockade model. And also the experimental strategy to directly access the topological loop operators measurement and the realization of topological boundary conditions. Please see the original paper if interested https://arxiv.org/abs/2011.12310.

INTRODUCTION

The physical realization of $\mathbb{Z}_2$ topological order as encountered in the paradigmatic toric code has proven to be an elusive goal. This paper shows that this phase of matter can be created in a two-dimensional array of strongly interacting Rydberg atoms. One can find a topological quantum liquid (TQL) as evidenced by multiple measures including: (i) a continuous transition between two featureless phases, (ii) a topological entanglement entropy of ln2 as measured in various geometries, (iii) degenerate topological ground states. Moreover, one can directly access the topological loop operators of this model, which can be measured experimentally using a dynamic protocol, providing a "smoking gun" experimental signature of the TQL phase. Finally, the paper shows how to trap an emergent anyon and realize different topological boundary conditions.

RYDBERG BLOCKADE MODEL

We consider hardcore bosons on the links of the kagome lattice with a two-dimensional version of the Fendley-Sengupta-Sachdev model:

$$H = \frac{\Omega}{2}\sum_i (b_i + b_i^\dagger) - \delta \sum_i n_i + \frac{1}{2}\sum_{i,j} V(|i-j|) n_i n_j \quad (0.1)$$

We set $\Omega > 0$ .For Rydberg atoms, $V(r) \sim \frac{1}{r^6}$. Here we conseder a simpler blockade in a particular disk:

$$V(r) = \begin{cases} +\infty & if \quad r \le 2a \\ 0 & if \quad r > 2a. \end{cases} \quad (0.2)$$

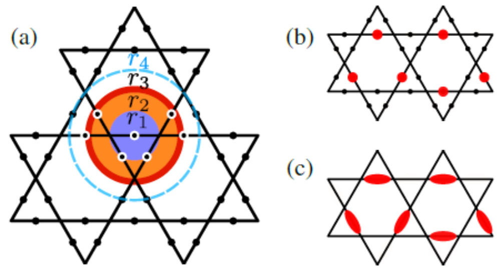


FIG. 1: Rydberg blockade model and relation to dimer model. (a) Hardcore bosons on the links of the kagome lattice (forming the ruby lattice) are strongly-repelling, punishing double-occupation within the disk $r \le r_3 = 2a$. (b) An example of a state consistent with the Rydberg blockade at maximal
lling. (c) Since the blockade forbids occupation of any two touching bonds, we can equivalently draw the configuration as a dimer covering on the kagome lattice.

Here the lattice spacing a is the shortest distance between two atoms. As shown in Fig.1(a), with this interaction range, a given site is coupled to six other sites, which are ordered in pairs at distances $r_1 = a, r_2 = \sqrt{3}a$ and $r_3 = 2a$. The Rydberg blockade implies that any two sites within this distance cannot both be occupied (Fig. 1(b)), which we can interpret as a dimer state on the kagome lattice if the system is at maximal filling (see Fig. 1(c)).

PHASE DIAGRAM

We now study the phase diagram of the model in Eq.(0.1) with the blockade in Eq.(0.2) using the density matrix renormalization group (DMRG). One can explicitly ecgorce $V(r_1) = +\infty$ and $V(r_2) = V(r_3) = 50\Omega$ by working in the reduced Hilbert space where each triangle of the kagome lattice (containing three atoms) only has four states: empty or a dimer on one of the three legs. When $\frac{\delta}{\Omega}$ is low enough, the system is adiabatically connected to the empty state and is thereby completely trivial. For very large $\frac{\delta}{\Omega}$ we enter the regime that is perturbatively described by a dimer model.Its ground state spontaneously breaks crystalline symmetries and forms a valence bond solid (VBS). Remarkably, for intermediate $\frac{\delta}{\Omega}$, these two phases are separated by another featureless phase, as shown in Fig.2 by the diverging correlation length ξ and the entanglement entropy S between two rings of the cylinder. This intermediate phase can be identified as a $\mathbb{Z}_2$ spin liquid by comparing the entanglement entropy.

One characteristic feature of topological phases of matter can be found in the scaling of the entanglement entropy. Gapped phases of matter satisfy an area law: for a region with perimeter L, we have $S(L) = \alpha L - \gamma$. For a $\mathbb{Z}_2$ spin liquid, $\gamma = ln2$. We take a point in the middle of the presumed spin liquid say $\frac{\delta}{\Omega} = 1.7$, and numerically obtain the entanglement entropy upon bipartitioning the infinitely long cylinder in two halves. Doing this for different circumferences, one extracts $\gamma \approx ln2$ as shown in Fig.3. For comparison, for a point in the trivial phase ($\frac{\delta}{\Omega} = 1$), one can find $\gamma \approx 0$.

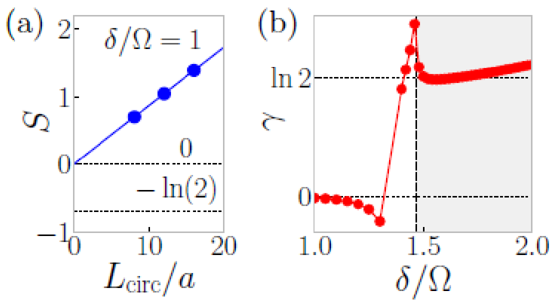


FIG. 3: Topological entanglement entropy.

STRING OPERATORS AND ANYON CONDENSATION

The two string operators are the 't Hooft line $e^{i\pi \int E}$ and the Wilson line $e^{i \int A}$ which anticommute at intersection points. The string operator $e^{i\pi \int E}$ corresponds to the parity of dimers along a string. To be precise, we define its action on a single triangle in Fig.4(a) (orange dashed line); we refer to this diagonal parity string as P. In the dimer basis, the dual string $e^{i \int A}$ has to be off-diagnal, such a string has a well defined action on single triangles, as shown in Fig.4(a) (solid blue line); we refer to this string as Q. The electric e and magnetic m excitations of this $\mathbb{Z}_2$ lattice gauge theory live at the endpoints of the Q and P strings, respectively. The spin liquid is defined by the deconfinement of these excitations. The nearby phases correspond to condensing either the e or the m, which respectively condenes m or e due to the mutual statistics. These condensates can be diagnosed by the open P or Q strings attaining long-range order. Generically these strings will decay to zero since the ground state has virtual e and m fluctuations. For this reason, Bricmont and Frohlich and Fredenhagen and Marcu independently introduced the normalized string operator in Fig.4(c), which we will refer to as the BFFM string order parameter. Evaluating the BFFM string order parameter for the Rydberg Blockade model, we see that Q only has long-range order in the trivial phase corresponding to an e-condensate whereas P only has long-range order in the VBS phase corresponding to an m-condensate. In the intermediate spin liquid, both BFFM order parameters decay to zero, consistent with the claim that this is the deconfined phase of the lattice gauge theory.(As shown in Fig.5).

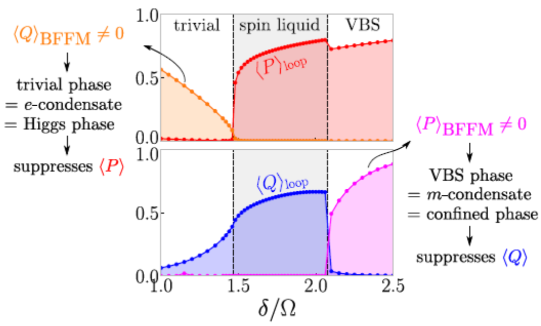


FIG. 5: Diagnosing phases in terms of topological string operators. One can find that the trivial phase is an e-condensate and the VBS phase is an m-condensate.

MEASURING AN OFF-DIAGONAL STRING BY TRANSFORMING IT INTO A DIAGONAL STRING

By measuring the string opertors introduced above, one can identify the spin liquid and its nearby phases. The parity string P can be straight forwardly measured in the lab since it is diagonal in the occupation basis and can be read off from the snapshots of the Rydberg states. The off-diagonal string Q is more challenging to measure directly. Actually by time-evolving with a quenched Rydberg Hamiltonian, it becomes a diagonal observable, making it experimentally accessible.

$$H' = \frac{\Omega}{2} \sum_i (ie^{i\alpha} b_i^\dagger + h.c.) - \delta \sum_i n_i + \frac{1}{2} \sum_{i,j} V(|i-j|) n_i n_j \tag{0.3}$$

It is thus sufficient to consider a single triangle, and by writing the P and Q operators defined in Fig.4(a) as 4×4-matrices acting on the Hilbert space of a single triangle, one straight forwardly derives:

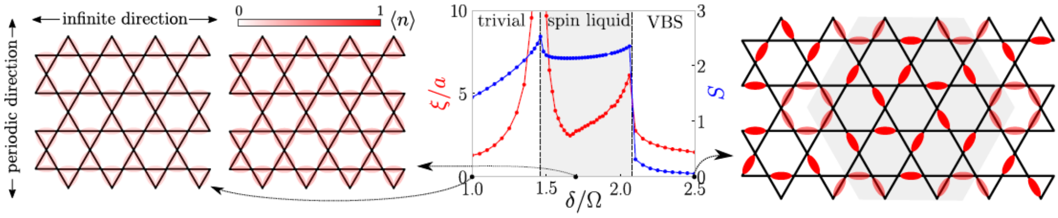


FIG. 2: Phase diagram of Rydberg blockade model on the links of the kagome lattice.

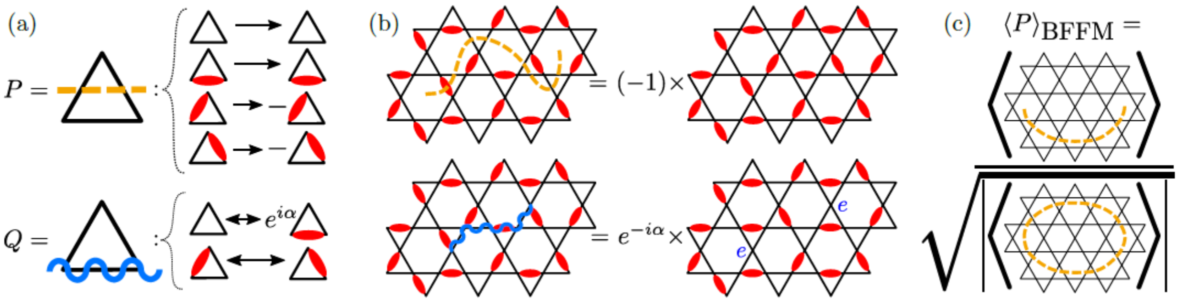
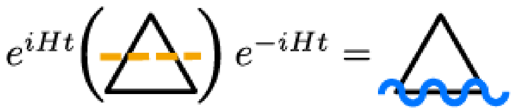


FIG. 4: Topological string operators. (a) The two different string operators are defined by their action on a single triangle. We call the diagonal and off-diagonal string operators P and Q, respectively. (b) An example of the action of the string operators on a classical dimer state. (c) The defination of the BFFM order parameter



Thus, one can effectively measure Q along a string by first time-evolving with H' and then measuring the P string on the resulting state.

CREATING TOPOLOGICAL DEGENERACY ON THE PLANE

There are two topologically-distinct boundary conditions for a $\mathbb{Z}_2$ spin liquid. These are characterized by whether the e or m anyon condenses at the edge. If one interprets a boundary as a spatial interface from the topological phase to a non-topological phase, it is natural that the characterization of the nearby phases carries over to describe boundary conditions. Similarly, these e and m condensates along the boundary can be diagnosed using the string operators introduced above.

One can numerically determine the resulting boundary phase diagram for the blockade model on an infinitely-long strip geometry, where we choose the bulk to be deep in the spin liquid at $\frac{\delta}{\Omega} = 1.7$. The results are shown in Fig.6. In line with the above expectation, we see that before we change the boundary detuning, i.e., $\delta_{bdy} = \delta$ the strip realizes an m-boundary. As we reach $\delta_{bdy} \approx 0.5\delta$there is a boundary phase transition (where the correlation length diverges along the infinite direction) after which the parity string dies out, making way for a strong signal for the Q string. In this regime, we stabilize the e-boundary. As we further decrease $\delta_{bdy} \to 0$, we are effectively removing these links from the model, with the remaining geometry again spontaneously realizing an m-boundary. Using the knowledge of the above boundary phase diagram, it is now straightforward to construct a rectangular geometry with a topological ground state degeneracy. A schematic picture is shown in Fig.7: a square slab where the four boundaries are alternatingly e- and m-condensed.

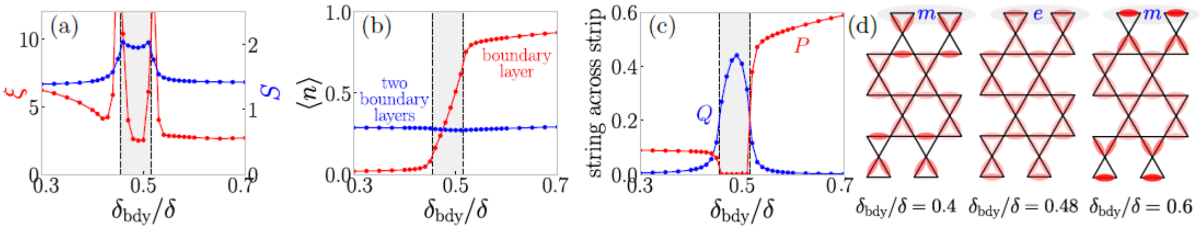


FIG. 6: Boundary phase diagram of the blockade model. The bulk is the spin liquid at $\frac{\delta}{\Omega} = 1.7$, but we tune δ_{bdy} on the outermost boundary links.

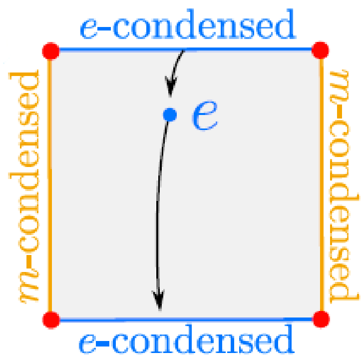


FIG. 7: Alternating e- and m-condensed boundaries imply a twofold degeneracy

OUT LOOK

The paper has demonstrated that Rydberg blockade on the ruby lattice can be utilized to stabilize a $\mathbb{Z}_2$ spin liquid. Specifically, the theoretical predictions outlined above can be probed using programmable quantum simulators based on neutral atom arrays trapped in optical tweezer arrays. In particular, the required atom arrangements can be realized using demonstrated atom sorting techniques, while relevant effective blockade range can be readily implemented using laser excitation into Rydberg states with large principle quantum number $60 < n < 100$. The spin liquid phase can be created via adiabatic sweep of laser detuning, starting from the disordered phase to a desired value of positive detuning, as demonstrated previously for one-dimensional systems. Potentially, these systems can explored for the realization of topologically-protected quantum bits, with an eye towards developing new, robust approaches to manipulating quantum information.

Acknowledgements

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