Physics 215B: QFT part two

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OH: after lecture ...

Work: weekly psets. 1st HW due Mon Jan 10 11:59 pm.

Media: Zoom until ... ?

Introductory Remarks & Goals:

QFT = QM of extensive d.o.f.

$$\begin{cases} \mathcal{H} = \bigotimes_{x \in \text{space}} \mathcal{H}_x \\ H = \sum_{\text{patch}} H\left(\begin{smallmatrix}\text{nearby} \\ \text{patches}\end{smallmatrix}\right) \end{cases}$$

(like Sod)

MICRO

$\Big\downarrow$ RG

macro

__Continuum__: (easier to compute.)

$\rightsquigarrow$ __UV divergences__

Results for physics better not depend on how we regulate them.

Role for RG: explain why.

__Principle__: express predictions in terms of observables.

__Goals__: • coarse-graining in QFT.

$$
\begin{array}{ccc}
UV & \text{grass + dirt} & \\
\Big\downarrow{\scriptstyle RG} & \Big\downarrow & \\
IR & \text{sod} & UV \\
 & \Big\downarrow & \Big\downarrow \\
 & \text{lawn} & IR
\end{array}
$$

- "non-renormalizable" $\neq$ "bad."
  $\longrightarrow$ Effective field theory.

- non-Abelian gauge theory.

- beyond pert. theory.

  ⋮

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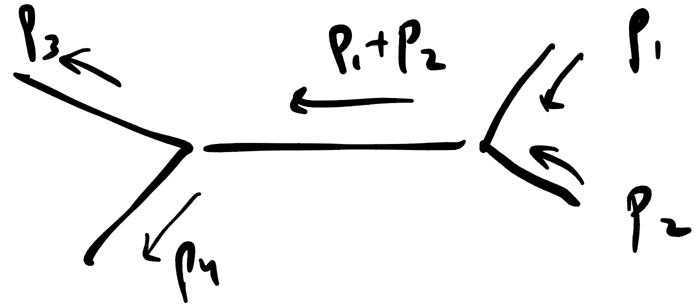
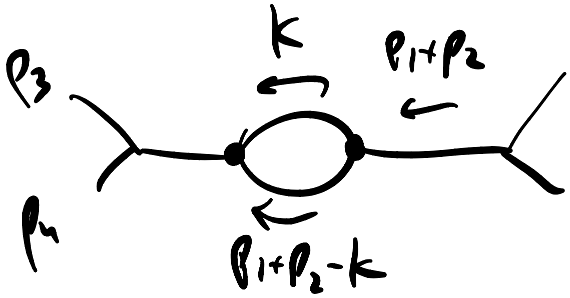
1. To infinity & beyond

Leading order = tree diagrams (no loops)



= classical.

vs :



$\propto \int d^D k \, (\ldots)$

1.1 Violation of scale invariance in QM

$$\mathcal{L}_{QED} \sim F^2 + \bar\Psi(\not{D} + m)\Psi$$

$$\not{D} = (\partial + eA)_\mu \gamma^\mu$$

$$4 = [F^2] = [\bar\Psi \not{\partial} \Psi]$$

$$\Rightarrow [A] = 1, \quad [\Psi] = 3/2 \quad \Rightarrow [e] = 0.$$

$$\mathcal{L}_{\phi^4} = (\partial\phi)^2 + m^2\phi^2 + r\phi^4$$

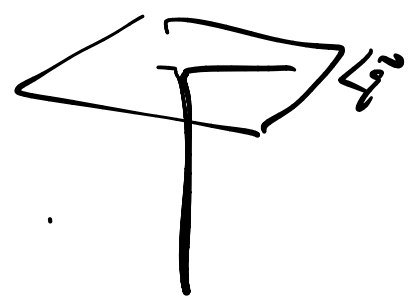
$\underline{D=4}$: $[\partial\phi]^2 = 4 \Rightarrow [\phi] = 1 \Rightarrow [r] = 0.$

$\Rightarrow$ (classically) scale invariant.

$$S[\vec{q}] = \int dt \left(\frac{\dot{q}^2}{2} - V(q) \right) \qquad (\vec{X} = \sqrt{m}\,\vec{q}.)$$

$$V(\vec{q}) = -g_0 \underbrace{\delta^{(2)}(\vec{q})}_{= \delta(q_x)\delta(q_y)}.$$

$(g_0 > 0.)$



$\hbar = 1$ $\qquad [t] = \left[\left(\frac{\text{energy}}{\hbar}\right)^{-1}\right] = -1$

$$[dt] = -1$$

$$0 = [S] = [\hbar] \Rightarrow 0 = \left[\int dt\, \dot{q}^2\right]$$

$$\Rightarrow [q] = -\frac{1}{2}$$

$$1 = \int dq\, \delta(q)$$

$$\left([\phi] = \frac{D-2}{2}\right)$$

w/ $D = 0+1$

$$\Rightarrow 0 = [dq] = -[\delta(q)]$$

$$[\delta^d(q)] = -[q]d = \frac{d}{2} \Rightarrow [\delta^2(q)] = 1.$$

$$\Rightarrow \text{in } d=2 \quad [g_0] = 0.$$

(classically) scale invariant.

QM.

$$H = \frac{p_x^2 + p_y^2}{2} + V(q) \underset{\text{on } \psi(q)}{=} -\frac{1}{2}(\partial_x^2 + \partial_y^2) - g_0\,\delta^2(q)$$

$$\Rightarrow \left(-\frac{\hbar^2}{2}\nabla^2 + V(q)\right)\psi_E(q) = E\,\psi_E(q)$$

let $\psi(\vec{q}) \equiv \int d^2p\, e^{i\vec{p}\cdot\vec{q}} \varphi(\vec{p})$

$$\delta^{(2)}(q) = \int d^2p\, e^{i\vec{p}\cdot\vec{q}} \cdot 1 \quad *$$

$$\Rightarrow \int d^2p\, e^{ip\cdot q} \left(\frac{p^2}{2} - E \right) \varphi(p) = g_0 \delta(q) \psi(0)$$

$$\overset{*}{=} g_0 \left(\int d^2p\, e^{ip\cdot q} \right) \psi(0)$$

$$\xRightarrow{\int d^2q\, e^{-ip\cdot q}(\text{RHS})} \quad \boxed{\left(\frac{p^2}{2} - E \right) \varphi(\vec{p}) = g_0 \underbrace{\int d^2p' \varphi(p')}_{\psi(0)}}$$

Two cases: ① if $\psi(0)$

$$\Longrightarrow \left(\frac{p^2}{2} - E \right) \varphi(p) = 0$$

$E = p^2/2 > 0$, $\psi \propto \sin p_x x \; e^{\pm i p_y y}$.

scattering states.

② $\psi(\bar{q}=0) \equiv \alpha \neq 0.$

$$\Longrightarrow \quad \frac{p^2}{2} - E \neq 0$$

$$\Longrightarrow \quad \varphi(\vec{p}) = \frac{g_0}{p^2/2 - E}\,\alpha \qquad ☆$$

$$\text{w/} \quad \alpha = \int d^2p'\,\varphi(\vec{p}\,')$$

Look for boundstates: $E = -\epsilon_B < 0$.

$\int (\text{RHS of } ☆)\, d^2p$

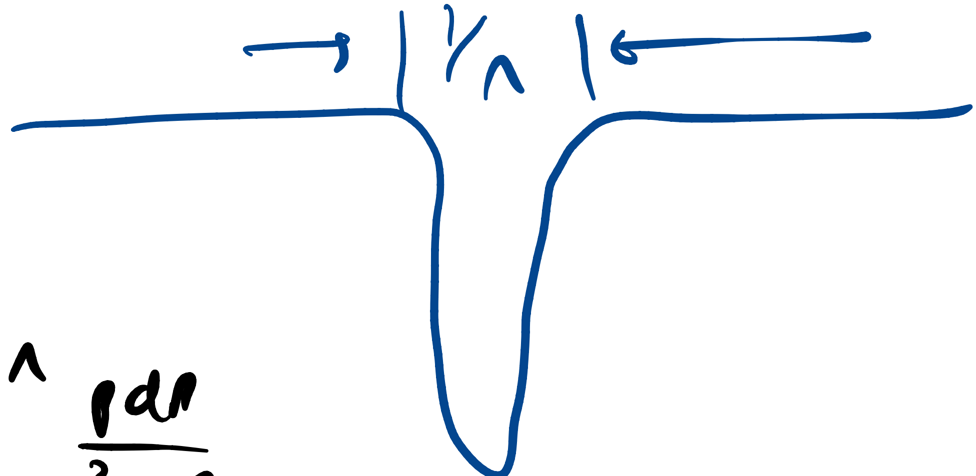
$$\Longrightarrow \quad 0 = \underbrace{\int d^2p\,\varphi(p)}_{=\alpha \neq 0}\left(1 - \int d^2p\,\frac{g_0}{p^2/2 + \epsilon_B}\right)$$

$$\Longrightarrow \quad \boxed{1 = \int d^2p\,\frac{g_0}{p^2/2 + \epsilon_B}}$$

$$\sim \int_\mu^\Lambda \frac{d^2p}{p^2} \sim \log \Lambda/\mu$$

<u>To regulate:</u> $\delta^2(q) = \int d^2p \, e^{ip\cdot q} \, 1$

$$\rightsquigarrow \int^{\Lambda} d^2p \, e^{ip\cdot q}$$



$$\int^{\Lambda} \frac{d^2p}{p^2/2+\epsilon_B} = 2\pi \int_0^{\Lambda} \frac{p\,dp}{p^2/2+\epsilon_B}$$

$$= 2\pi \log\left(1+\frac{\Lambda^2}{2\epsilon_B}\right). \qquad (\Lambda \gg \epsilon_B)$$

$$1 \overset{!}{=} g_0 \int (\downarrow) = \frac{g_0}{2\pi\hbar^2} \log\left(1+\frac{\Lambda^2}{2\epsilon_B}\right)$$

- only has soln when $g_0 > 0$. (attractive potential) ✓

Solve for ϵ_B :

$$\epsilon_B = \frac{\Lambda^2}{2} \frac{1}{e^{2\pi\hbar^2/g_0^2} - 1} \xrightarrow[\text{fix } g_0]{\Lambda \to \infty} \infty$$

SACRED FICTION FICTION

Suppose : we know $S[q]$ is a good model of our system & we measure ϵ_B.

solve for $g_0(\Lambda)$:

$$g_0(\Lambda) = \frac{2\pi\hbar^2}{\log\left(1 + \Lambda^2/2\epsilon_B\right)}$$

$\Longrightarrow$ bare coupling $g_0(\Lambda)$ must be cutoff dependent!

Another example: $V(q) = \frac{\lambda}{q^\alpha}$ (Try $\alpha > 2$.)

$[q] = -1/2$ $\quad [\lambda] = 1 - 2\alpha$ $\Rightarrow \alpha = 2$ is dim'less.

"dimensional transmutation"

Instead of a dim'less coupling constant g_0 a family of theories labelled by

$\rightsquigarrow$ a fam. of theories labelled by a **dimensionful** qtty ϵ_B.

(choose units w/ $\epsilon_B = 1$)

$\rightarrow$ only one theory.

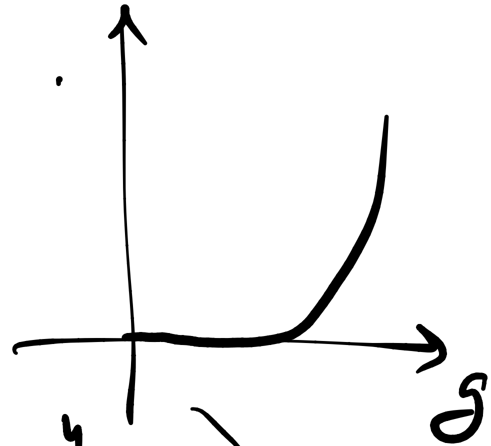
$$\epsilon_B = \frac{\Lambda^2}{2} \frac{1}{e^{2\pi\hbar^2/g_0} - 1} \overset{g_0 \to 0}{\simeq} \underline{\underline{e^{-2\pi\hbar^2/g_0}}}$$

$$\neq \sum_{n=0}^{\infty} g_0^n f_n(\Lambda)$$

non-analytic in g_0 at $g_0 = 0$

can never see in perturbation theory!

$$\left(\frac{\partial}{\partial g}\right)^n \left(e^{-\frac{1}{g}}\right)\Big|_{g=0} = 0.$$



(as in $I(g) = \int dq\, e^{-q^2+gq^4}$)

$$\epsilon_B \underset{g_0 \ll 1}{\cong} \frac{\Lambda^2}{2} e^{-2\pi\hbar^2/g_0} = 10^{-28}\cdot\Lambda^2$$

$g_0 = .1$

$$g_0(\Lambda) = \frac{2\pi\hbar^2}{\log\left(1+\frac{\Lambda^2}{2\epsilon_B}\right)} \xrightarrow{\Lambda^2 \gg \epsilon_B} \frac{2\pi\hbar^2}{\log \Lambda^2/2\epsilon_B} \xrightarrow{\Lambda^2 \gg \epsilon_B} 0.$$

"Asymptotic freedom"

bare coupling is weak when cutoff is removed.