$$\bar{q}_\alpha \quad m_\alpha \quad q_\alpha$$

$$\alpha \xrightarrow{\qquad\qquad} \beta \; = \; \delta_{\alpha\beta} \, \frac{i}{\not{k} - m_\alpha}$$

Last time: $\dfrac{\partial}{\partial \log\mu} g = \beta(g) = \underline{\underline{-}}\, C \dfrac{g^3}{16\pi^2} + O(g^4)$

$$C = \frac{11}{3} N - \frac{2}{3} N_f \qquad \beta < 0 \text{ if } N_f < 6N \atop = 18$$

$$\frac{\partial}{\partial \log\mu} g = - C \frac{g^3}{16\pi^2}$$

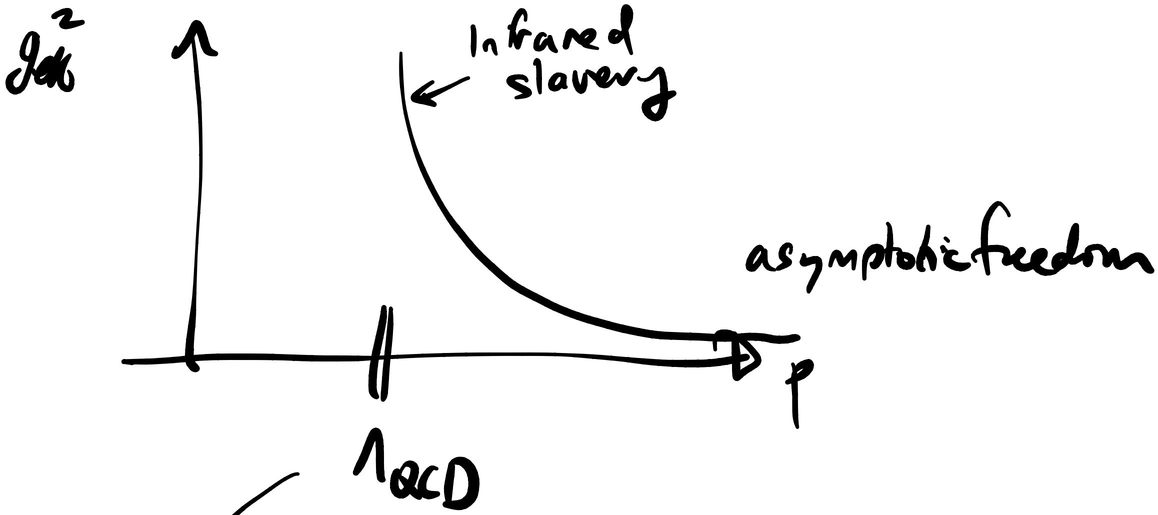
solve: $\dfrac{dg}{g^3} = - \dfrac{C}{16\pi^2} \, d\log\mu$

$$\Rightarrow \quad -\frac{1}{g^2} \Big|_M^{\sqrt{|p^2|} = p} = -\frac{C}{16\pi^2} \log\frac{p}{M}$$

$$\Rightarrow g^2_{\text{eff}}(p) = \frac{g_0^2}{1 + \frac{g_0^2}{16\pi^2} C \log\frac{p}{M}}$$

Contrast: in QED:

$$e^2(p) = \frac{e_0^2}{1 - \frac{e_0^2}{16\pi^2} \# \log\frac{p}{M}}.$$



def'd by $\quad g_{eff}^2(p=\Lambda_{QCD}) \sim 1.$

An intuitive picture of anti-screening (?)

$N_f = 0$ (quarks screen)

$N = 2.$ $\quad f^{abc} = \epsilon^{abc}.$

1
2
3

choose Coulomb gauge $\partial_i A^{ia} = 0$

- no ghosts
- no A_0, $\hat{A}\cdot\hat{k}$
- no Lorentz.

$$E^{ia} \equiv F^{0ia}$$

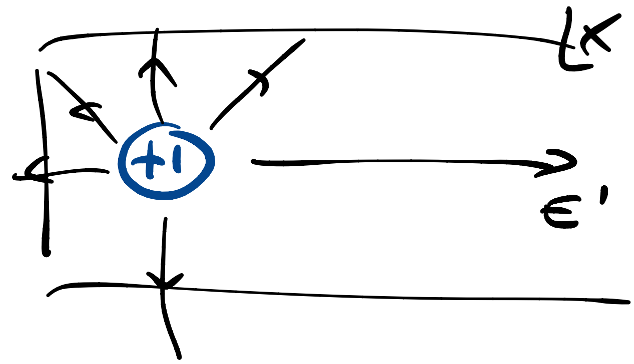
$$0 = \frac{\delta S}{\delta A_0^a} \Rightarrow g\rho^a = D_i E^{ia} = \partial_i E^{ia} + g f^{abc} A_i^b E^{ic}$$

Insert a static color charge $\rho^a(x) = \delta^3(\vec{x})\delta^{a1}$

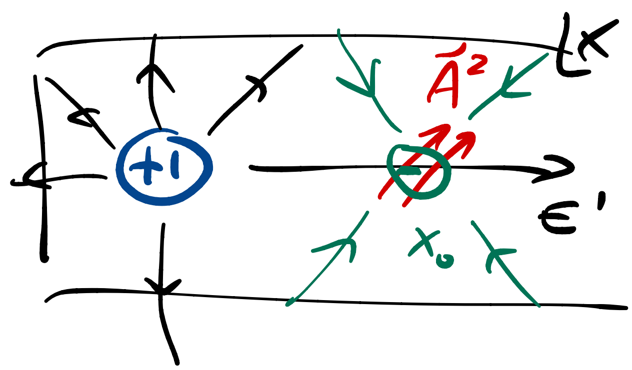
$$\underline{\partial\cdot E^{ia} = g\,\delta^{(3)}(x)\,\delta^{a1}} + g\,\epsilon^{abc}\,\underline{A^{bi}E^{ic}}$$

assume g small, perturbatively: $E \sim g$

① $\vec{E}^a(x) = g\,\delta^{1a}\,\dfrac{\hat{x}}{x^2}$



② $\underline{\text{QM}}$: suppose a fluctuation of $\vec{A}^{b=2}(x)$



③ $g\,\epsilon^{abc}A^{bi}E^{ic}$

$$= g\,\underbrace{\epsilon^{321}}_{=-1}\,\underline{\vec{A}^2\cdot\vec{E}^1}$$

④ This source produces

$$\vec{E}^3(x) \propto -\frac{\vec{x}-\vec{x}_0}{|x-x_0|^3}$$

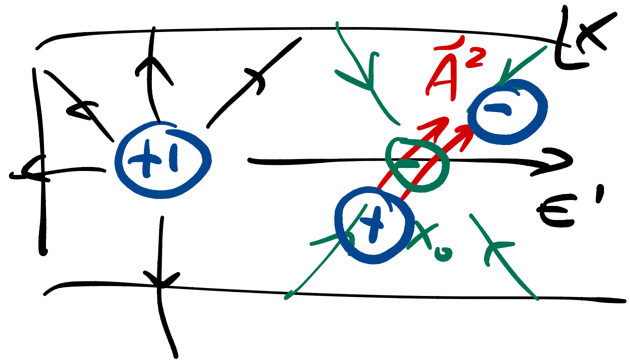
⑤ $\vec{\nabla} \cdot \vec{E}^1 = \dots + g \underbrace{\epsilon^{123}}_{=+1} \vec{A}^2 \cdot \vec{E}^3$

Source for E^1 where $A^2 \parallel E^3$

Sink for E^1 " $-$

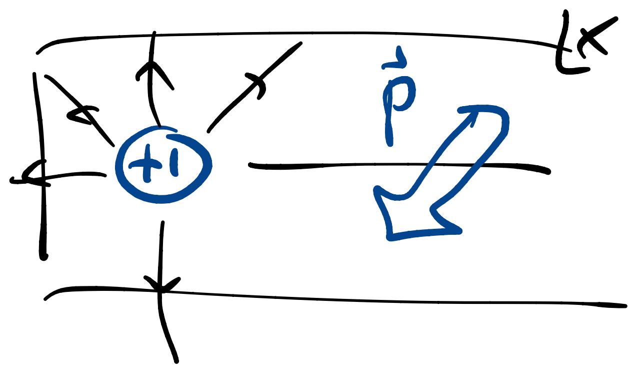
→ dipole of E^1 charge

$\vec{p} \propto -\vec{A}^2$

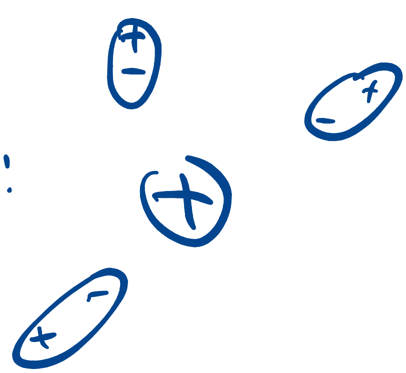


dipole can point TOWARDS the source.

antiscreening!



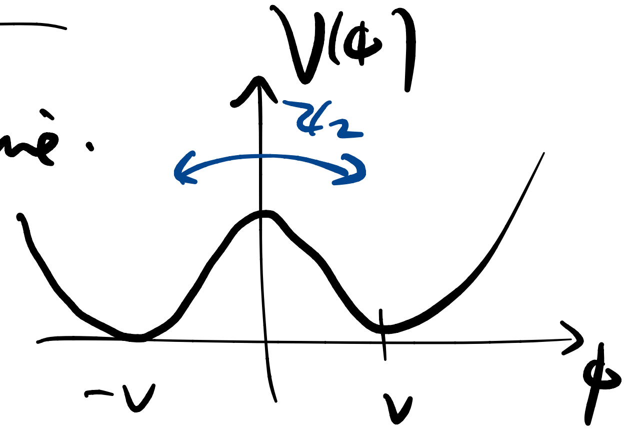
screening:



6 Renormalization Group, briefly

6.1 Wilsonian perspective

ϕ^4 thy in eucl. spacetime.



$V(\phi) = -|m|^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \dots$

$\mathbb{Z}_2 : \phi \to -\phi$

like a $\boxed{\text{ferromagnet}}$

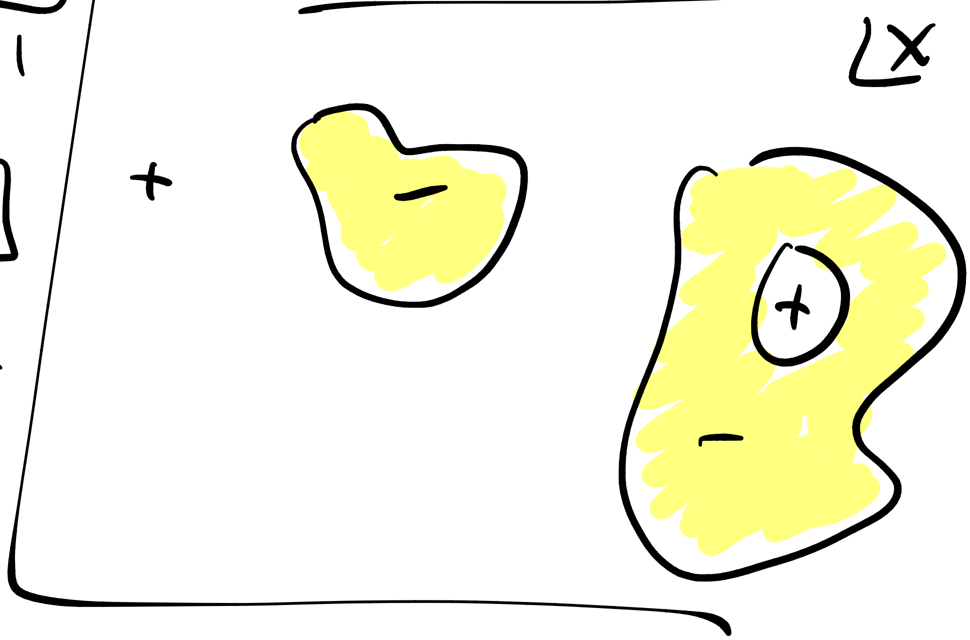
Ising model $\sigma^i = \pm 1$

$J\sum_{\langle ij\rangle}\sigma^i\sigma^j$ $\quad \boxed{J>0}$

penalizes domain walls

like $\mathcal{L} \ni (\partial\phi)^2$

A config of ϕ:



$\underline{\underline{\sigma^i = \pm 1}}$ $\qquad V(\phi)$ penalizes $\phi \neq \pm v$.

plausible: the critical pt where $m^2 \to 0$ is the Ising critical pt.

$$Z_\Lambda = \int_\Lambda [D\phi]\, e^{-\int d^Dx\, \mathcal{L}(\phi)}$$

$\int_\Lambda \equiv$ integrate over $\phi(x) = \int d^Dk\, e^{ikx}\phi_k$

w/ $\phi_k = 0$ for $|k| = \sqrt{\sum^D k_i^2} > \Lambda$

idea: Do the integrals of the high-energy modes first.

$$\phi(x) = \int d^Dk\, e^{ikx}\phi_k = \underbrace{\int dk\, e^{ikx}\phi_k}_{0<|k|<\Lambda-\delta\Lambda} + \underbrace{\int dk\, e^{ikx}\phi_k}_{\Lambda-\delta\Lambda<|k|<\Lambda}$$

$$\int \phi_>(x)\,\phi_<(x) = 0$$

$\equiv \phi^<$
smooth
slow
light

$\equiv \phi^>$
wiggly
fast
heavy.

$$Z_\Lambda = \int_{\Lambda-\delta\Lambda} [D\phi^<]\, e^{-\int d^Dx\, \mathcal{L}(\phi^<)} \underbrace{\int [D\phi^>]\, e^{-\int d^Dx\, \mathcal{L}_1(\phi^>,\phi^<)}}_{e^{-\int \delta\mathcal{L}(\phi^<)}}$$

result of interactions

$$= \int_{\Lambda-\delta\Lambda} [D\phi^<]\, e^{-\int [\mathcal{L}(\phi^<) + \delta\mathcal{L}(\phi^<)]} = e^{-S_{\rm eff}[\phi^<]}$$

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial\phi)^2 + \underline{\sum_{n \text{ even}}} g_n \phi^n + \underline{\underline{\;\cdots\;}} \quad (*)$$

all possible terms consistent w/ symmetries (like $\phi \to -\phi$, Lorentz ...)

like $(\partial^2\phi)^2$

$$\int d^Dx\, \mathcal{L}_1(\phi^<, \phi^>) = \int d^Dx \left(\frac{1}{2}(\partial\phi^>)^2 + \frac{1}{2}m^2(\phi^>)^2 + \# g_4\, \phi^< (\phi^>)^3 + \cdots + g_4 (\phi^>)^4 \right)$$

scalar field $\phi^>$ in a slowly-varying bg. scalar field $\phi^<$

what's $\delta\mathcal{L}(\phi^<)$? It must be of the form $(*)$

$S_{eff}[\phi^<] = \int (\mathcal{L} + \delta\mathcal{L})$ is of the same form w/ $g_n \to g_n + \delta g_n$

step 2: change units to make $\int_{\Lambda - \delta\Lambda}$ into $\int_\Lambda$

$\underline{\Lambda - \delta\Lambda \equiv b\Lambda \quad b < 1}$

$$\int_{\Lambda - \delta\Lambda} = \int_{\Lambda b}$$

$k = bk'$ $\quad |k'| < \Lambda.$ $\qquad x = x'/b \quad \partial' = \frac{\partial}{\partial x'} = \frac{\partial_x}{b}$

$$e^{ikx} = e^{ik'x'}$$

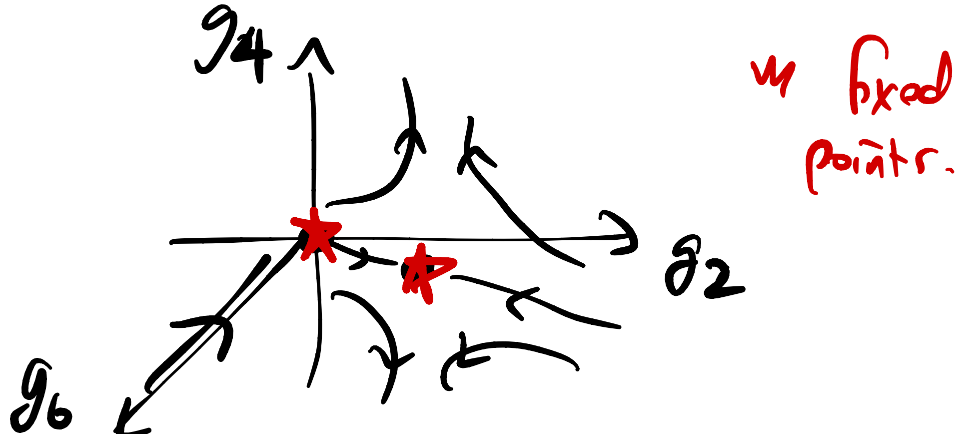
$$\int d^D x\, \mathcal{L}_{eff}(\phi^<) = \int d^D x'\, b^{-D} \Big(\tfrac{1}{2} \underline{\underline{b^2}} (\partial' \phi^<)^2 + \sum_n (g + \delta g_n)(\phi^<)^n + \dots \Big)$$

let $\phi' \equiv b^{\frac{2-D}{2}} \phi^<$

$$= \int d^D x' \Big[\tfrac{1}{2} (\partial' \phi')^2 + \sum_n (g_n + \delta g_n) b^{-D + n\frac{(D-2)}{2}} (\phi')^n + \dots \Big]$$

<u>end result</u>: $\quad g_n' = \underline{\underline{b^{n\frac{(D-2)}{2} - D}}} (g_n + \delta g_n)$

<u>RG flow in the space of couplings</u>:



<u>eg</u>: ignore δg_n. $\underline{b < 1}$. g_n w/ $n\frac{(D-2)}{2} - D > 0$ get smaller.

<u>IRRELEVANT</u>.

In D $\frac{n(D-2)}{2} - D < 0$ get bigger as $b \to 0$

Relevant

eg $n = 2$ $\underline{(m')^2 = b^{-2} m^2}$.

So far: this counting is the <u>same</u> as dimensional analysis.

w/ the $\delta\mathcal{L}$ term $\xrightarrow[\text{particularly}]{}$ different.

for $\frac{n(D-2)}{2} - D = 0$

(marginal)

---

pert. thy can be controlled $\underline{D = 4 - \epsilon}$.

eg: Ising $\left(\begin{array}{c}\text{a scalar w/} \\ \phi \to -\phi \text{ sym}\end{array}\right)$

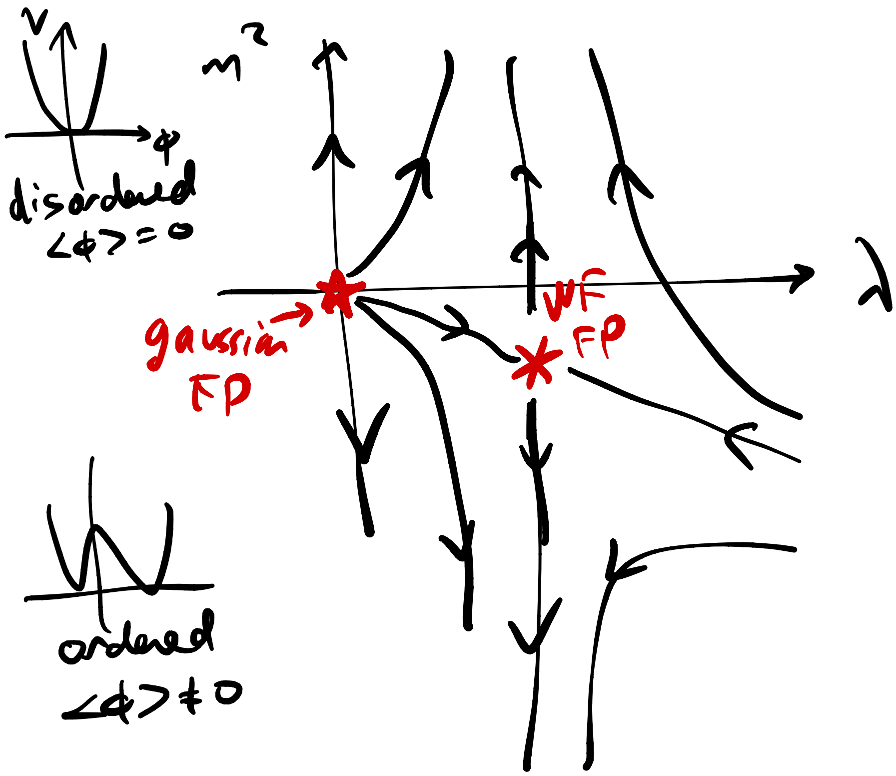
$$-\beta_\lambda = \underbrace{\epsilon\lambda}_{\text{engineering}} - \underbrace{a\lambda^2}_{\text{interacting}} + O(\lambda^4)$$

(high-energy p f'n)

$a > 0$

has a zero for $\underline{\lambda = \epsilon/a}$. Wilson-Fisher fixed pt.

arrows point toward IR.



DRAGONS

??

rates of departure from fixed pt $\longleftrightarrow$ critical exponents

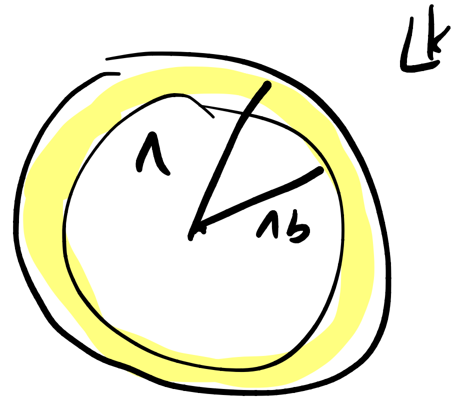
eg: rate of m^2 term $\longleftrightarrow$ correlation length critical exponent.

same story w/ N scalar fields & O(N) symm.

Lessons: • Wilson RG = doing the integrals a little at a time.

• elimination of modes does not produce any singularities

• symmetries of the UV are symmetries of S_{eff}.



• $\beta_\lambda > 0 \longrightarrow$ "$\phi^4_{D=4}$ D.N.E."

" ... is trivial".

in the sense that: If valid to $\Lambda \to \infty$ $\lambda(\Lambda \to \infty) = \infty$.

in order to get $\lambda(0) > 0$.

Next wk: Tuesday 2pm

Wed 12:3 pM

$$e^{i\oint_c A} = e^{i\int d^D x \, J^\mu(x) A_\mu}$$

$$\oint_c A = \oint dt \, \frac{dx^\mu}{dt} A_\mu(x)$$

$\{x^\mu(t)\}$ is c

$$= \int d^4 y \int dt \, \frac{dx^\mu}{dt} A_\mu(y) \, \delta^4(x(t) - y)$$

$$= \int d^4 y \, A_\mu(y) \, J^\mu(x)$$

$$J^\mu(x) = \int dt \, \frac{dx^\mu}{dt} \, \delta^4(x(t) - y)$$