University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 213/113 Winter 2023
Assignment 1

Due 11:00am Tuesday, January 17, 2023

1. **Too many numbers.**

   Find the number of qbits the dimension of whose Hilbert space is the number of atoms in the Earth. (It's not very many.) Now imagining diagonalizing a Hamiltonian acting on this space.

2. **Warmup for the next problem.**

   Parametrize the general pure state of a qbit in terms of two real angles. A good way to do this is to find the eigenstates of

   $$\boldsymbol{\sigma}^n \equiv \check{n} \cdot \vec{\boldsymbol{\sigma}} \equiv n_x \mathbf{X} + n_y \mathbf{Y} + n_z \mathbf{Z}$$

   where $\check{n}$ is a unit vector.

   Compute the expectation values of $\mathbf{X}$ and $\mathbf{Z}$ in this state, as a function of the angles θ, φ.

3. **Mean field theory is product states.**

   Consider a spin system on a lattice. More specifically, consider the transverse field Ising model:

   $$\mathbf{H} = -J \left(\sum_{\langle x,y \rangle} Z_x Z_y + g \sum_x X_x \right) .$$

   Consider the mean field state:

   $$|\psi_{\text{MF}}\rangle = \otimes_x |\psi\rangle_x = \otimes_x \left(\sum_{s_x = \pm} \psi_{s_x} |s_x\rangle_x \right) , \tag{1}$$

   i.e., restrict to a product state where the state ψ of each spin is the same.

   Write the variational energy for the mean field state, *i.e.* compute the expectation value of $\mathbf{H}$ in the state $|\psi_{\text{MF}}\rangle$, $E(\theta, \varphi) \equiv \langle \psi_{\text{MF}}| \mathbf{H} |\psi_{\text{MF}}\rangle$.

   Assuming s_x is independent of x, minimize $E(\theta, \varphi)$ for each value of the dimensionless parameter g. Find the groundstate magnetization $\langle \psi | Z_x |\psi\rangle$ in this approximation, as a function of g.

4. **Classical versus quantum circuit sampling.** [This is an optional open-ended problem intended as food for thought.]

   We showed in lecture that the set of states reachable from a given state by polynomial-depth quantum circuits is a small fraction of the whole Hilbert space. This followed by close analogy with the statement that most boolean functions aren't computable using a polynomial number of gates. The closeness of this analogy leads to the following question:

   Let $P_C(s,t)$ be the probability of obtaining bit string s when starting with N uniform iid bits and feeding them through a classical circuit C made of t layers of 2-bit gates.

   Let
   $$P_U(s,t) = \left|\langle s^z = s|U \otimes_{i=1}^N |s^x = 1\rangle\right|^2$$
   where U is a quantum circuit made from t layers of neighboring 2-qbit gates. This is the probability distribution for measurements of σ_i^z on the state resulting from acting a quantum circuit U on a product of σ^x eigenstates.

   Show that when $t = 0$ the distributions are the same.

   Under some assumptions about the scaling of t with N, can we find a $P_U(s,t)$ that can never be a $P_C(s,t)$?

   If we were allowed to measure in the X-basis as well as the Z-basis then it would be easy, because we could for example just design the circuit to produce at time t Bell pairs between spins $2n-1$ and $2n$, and do exactly the Bell protocol on them.

   Warning: I don't know the answer.