University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2026
Assignment 4

Due 11:59pm Monday, February 2, 2026

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1. **Meson scattering.** Consider the Yukawa theory with fermions in $D = 3+1$, with
$$\mathcal{L} = \bar{\psi}\left(\mathbf{i}\partial\!\!\!/ - m\right)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + \mathcal{L}_{\text{int}}$$
and $\mathcal{L}_{\text{int}} = y\bar{\psi}\psi\phi$.

   (a) Consider the correction to the process $\phi\phi \to \phi\phi$ coming from a fermion loop. What counterterm is required to renormalize this interaction? (You don't need to actually do the integral for this problem.)

   (b) Do you need a cutoff-dependent counterterm of the form $\delta_3\phi^3$ in this theory? [Hint: Use symmetries.]

2. **Bosons have worse UV behavior than fermions.**

   Consider again the Yukawa theory with action
$$S[\phi,\psi] = -\int d^Dx\left(\frac{1}{2}\phi\left(\Box + m_\phi\right)\phi + \bar{\psi}\left(-\partial\!\!\!/ + m_\psi\right)\psi + y\phi\bar{\psi}\psi + \frac{g}{4!}\phi^4\right) + \text{counterterms}.$$

   (a) Show that the superficial degree of divergence for a diagram $\mathcal{A}$ with B_E external scalars and F_E external fermions is
$$D_{\mathcal{A}} = D + (D-4)\left(V_g + \frac{1}{2}V_y\right) + B_E\left(\frac{2-D}{2}\right) + F_E\left(\frac{1-D}{2}\right) \qquad (1)$$
   where V_g and v_y are the number of ϕ^4 and $\phi\bar{\psi}\psi$ vertices respectively.

   All the discussion below is about one-loop diagrams.

   (b) Draw the diagrams contributing to the self energy of both the scalar and the spinor in the Yukawa theory.

   (c) Find the superficial degree of divergence for the scalar self-energy amplitude and the spinor self-energy amplitude.

(d) In the case of $D = 3+1$ spacetime dimensions, show that (with a cutoff on the Euclidean momenta) the spinor self-energy is actually only logarithmically divergent. (This type of thing is one reason for the adjective 'superficial'.)

Hint: the amplitude can be parametrized as follows: if the external momentum is p^μ, it is

$$\mathcal{M}(p) = A(p^2)\not{p} + B(p^2).$$

Show that $B(p^2)$ vanishes when $m_\psi = 0$.

3. **Dimension-dependence of dimensions of couplings.**

(a) In what number of space dimensions does a four-fermion interaction such as $G\bar{\psi}\psi\bar{\psi}\psi$ have a chance to be renormalizable? Assume Lorentz invariance.

[optional] Generalize the formula (1) for $D_{\mathcal{A}}$ to include a number V_G of four-fermion vertices.

(b) If we violate Lorentz invariance the story changes. Consider a non-relativistic theory with kinetic terms of the form $\int dt d^d x \left(\psi^\dagger \left(\mathbf{i}\partial_t - D\nabla^2\right)\psi\right)$. (Here D is a dimensionful constant. In a relativistic theory we relate dimensions of time and space by setting the speed of light to one; here, there is no such thing, and we can choose units to set D to one.) For what number of space dimensions might the four-fermion coupling be renormalizable?

(c) In the previous example, the scale transformation preserving the kinetic terms acted by $t \to \lambda^2 t, x \to \lambda x$. More generally, the relative scaling of space and time is called the *dynamical exponent* z ($z = 2$ in the previous example). Suppose that the kinetic terms are first order in time and quadratic in the fields. Ignoring difficulties of writing local quadratic spatial kinetic terms, what is the relationship between d and z that gives scale-invariant quartic interactions? What if the kinetic terms are instead second order in time (as for scalar fields)?

4. **Scale invariance in QFT in $D = 0+0$, part 2.** [I got this problem from Frederik Denef.]

The story of $D = 0+0$ QFT is more interesting if there is more than one field, *i.e.* if we consider the statistical mechanics of a particle moving in more than one dimension. Consider the example of two degrees of freedom with Hamiltonian

$$H = \frac{1}{2}P_X^2 + \frac{1}{2}P_Y^2 + V(X,Y), \qquad V(X,Y) = aX^4 + bY^8 \tag{2}$$

for some nonzero constants a, b.

(a) This potential again has a scaling symmetry $V(\lambda^{1/4}X, \lambda^{1/8}Y) = \lambda V(X,Y)$. As a result, the model describes a fixed point, with constant heat capacity. Find the heat capacity.

(b) Restricting to deformations with independent symmetries under $X \to -X$ and $Y \to -Y$, and using the basic scaling properties of the deformations under the above scaling symmetry, what are the relevant, marginal and irrelevant deformations? (Note that in this case there are true marginal deformations that cannot be absorbed into the normalization of X and Y.)

(c) How does $\langle X^k Y^l \rangle$ depend on T at a fixed point satisfying $V(\lambda^{\Delta_X}X, \lambda^{\Delta_Y}Y) = \lambda V(X,Y)$?

A generic relevant deformation of (2) will flow to a Gaussian fixed point $V(X,Y) \sim X^2 + Y^2$ in the IR. Some other, more fine-tuned deformations will flow to other fixed points. For example, $\delta V(X,Y) = \epsilon Y^4$ will flow to $V(X,Y) = X^4 + Y^4$. But something more interesting happens for $\delta V(X,Y) = \epsilon X^2 Y^2$. We'll study this more on the next homework.