

University of California at San Diego – Department of Physics – Prof. John McGreevy  
**Physics 215B QFT Winter 2026**  
**Assignment 7**

Due 11:59pm Monday, February 23, 2025

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1. **Radiative corrections to Compton scattering.** Check that our prescription for renormalizing QED through one loop (e.g. using Pauli-Villars with renormalization conditions on the electron mass and the coupling  $\frac{e^2}{4\pi} = \frac{1}{137}$ ) suffices to remove all the cutoff dependence in the  $S$  matrix for Compton ( $e\gamma \rightarrow e\gamma$ ) scattering through  $\mathcal{O}(\alpha^2)$ .
2. **Yes, please, gauge invariance.** Verify for yourself that the one-loop vacuum polarization amplitude in QED (when computed using either the improved Pauli-Villars regulator or dim reg) satisfies the Ward identity, *i.e.* is proportional to  $q^\mu q^\nu - \eta^{\mu\nu} q^2$ . It's up to you how much of this to hand in.
3. **Yukawa couplings in QED.** Consider adding to QED an additional scalar field of (physical) mass  $m$ , coupled to the electron by

$$L_Y = \lambda \phi \bar{\psi} \psi.$$

Verify that the divergent contribution to the electron wavefunction renormalization factor  $Z_2$  from a virtual  $\phi$  equals the divergent contribution to the QED vertex  $Z_1$  from the one loop correction to the vertex with a virtual  $\phi$ .

For an added challenge, verify that the finite parts agree as well. [For purposes of matching the finite parts, some advice: we can put the electron lines on shell and sandwich between spinors satisfying the equations of motion (as we did for the QED vertex correction), and also set the incoming photon momentum  $q = p' - p = 0$ .]